Supplementary Material

[Proof of Proposition 1]

"If" part: assuming $\mathbf{L}$ c.p.d, $\forall c_1, \dots, c_n, c_{n+1}$ (with $c_{n+1} = -\sum_{i=1}^{n} c_i$)

$$
\begin{aligned}
0 &\leq \sum_{i,j=1}^{n+1} c_i c_j \mathbf{L}_{i,j} \\
&= \sum_{i,j=1}^{n} c_i c_j \, \mathbf{L}_{i,j} + c_{n+1} \sum_{i=1}^{n} c_i \, \mathbf{L}_{i,n+1} + c_{n+1} \sum_{j=1}^{n} c_j \, \mathbf{L}_{n+1,j} + c_{n+1}^2 \, \mathbf{L}_{n+1,n+1} \\
&= \sum_{i,j=1}^{n} c_i c_j \, \big(\mathbf{L}_{i,j} - \mathbf{L}_{i,n+1} - \mathbf{L}_{n+1,j} + \mathbf{L}_{n+1,n+1}\big) \\
&= \sum_{i,j=1}^{n} c_i c_j \hat{\mathbf{L}}_{i,j}.
\end{aligned}
$$

"Only If" part: assuming $\hat{\mathbf{L}}$ positive definite $\forall c_1, \dots, c_n$ (including when $\sum_i c_i = 0$) and for any u (for instance $u \in \{1, \dots, n\}$)

$$
\begin{aligned}
0 &\leq \sum_{i,j=1}^{n} c_i c_j \hat{\mathbf{L}}_{i,j} \\
&= \sum_{i,j=1}^{n} c_i c_j (\mathbf{L}_{i,j} - \mathbf{L}_{i,u} - \mathbf{L}_{u,j} + \mathbf{L}_{u,u}) \\
&= \sum_{i,j=1}^{n} c_i c_j \mathbf{L}_{i,j} - \sum_{i} c_i \mathbf{L}_{i,u} \sum_{j=1}^{n} c_j - \sum_{j=1}^{n} c_j \mathbf{L}_{u,j} \sum_{i=1}^{n} c_i + L_{u,u} \sum_{i=1}^{n} c_i \sum_{j=1}^{n} c_j \\
&= \sum_{i,j=1}^{n} c_i c_j \mathbf{L}_{i,j}
\end{aligned}
$$

■

		Binary	Binary × Gaussian													Multi-lap
			$10^{-6}\sigma$	$10^{-5}\sigma$	$10^{-4}\sigma$	$10^{-3}\sigma$	$10^{-2}\sigma$	$10^{-1}\sigma$	σ	10σ	$10^{2}\sigma$	$10^{3}\sigma$	$10^{4}\sigma$	$10^{5}\sigma$	$10^{6}\sigma$	
Unormalized	$k=1$	92.22	91.73	91.73	91.73	91.73	91.73	91.73	91.73	91.73	91.71	91.71	91.71	91.71	91.71	92.69
	$k=4$	88.90	87.95	87.95	87.95	87.95	87.95	87.95	87.95	87.95	87.95	87.92	87.92	87.92	87.92	89.61
	$k=32$	85.78	84.48	84.48	84.48	84.48	84.48	84.48	84.48	84.50	84.50	84.50	84.50	84.50	84.50	86.28
Normalized	$k=1$	92.34	91.78	91.78	91.78	91.78	91.78	91.78	91.75	91.75	91.75	91.75	91.75	91.77	91.77	92.78
	$k=4$	89.67	88.56	88.56	88.56	88.56	88.56	88.56	88.56	88.59	88.59	88.59	88.56	88.56	88.56	90.13
	$k=32$	87.60	86.48	86.48	86.48	86.48	86.48	86.48	86.48	86.50	86.50	86.50	86.50	86.50	86.50	88.17
Random w	$k=1$	92.57	91.17	91.17	91.17	91.16	91.17	91.17	91.17	91.20	91.20	91.20	91.17	91.17	91.17	92.88
	$k=4$	95.81	93.88	93.88	93.88	93.81	93.81	93.80	93.80	93.79	93.83	93.81	93.80	93.80	93.81	96.12
	$k=32$	95.77	93.85	93.85	93.85	93.85	93.85	93.86	93.86	93.86	93.84	93.84	93.84	93.84	93.84	96.10
Multi-lap		96.96	94.26	94.26	94.26	94.28	94.28	94.28	94.28	94.27	94.26	94.26	94.26	94.26	94.26	98.14

Table 5: Performances on SBU for different elementary laplacians (normalized, unormalized and random walk) and their marginal and total combinations using MLGCN (note that our expansion is not used). In this table, "binary" means that $\mathbf{A}^k$ is used to build the elementary laplacian while "binary × gaussian" means that "$\mathbf{A}^k \times$ gaussian similarity" is used instead; for each graph $\mathcal{G}$, the scale σ of the gaussian similarity is taken as the average distance between node features in $\mathcal{G}$.

		Binary	Binary × Gaussian													Multi-lap
			$10^{-6}\sigma$	$10^{-5}\sigma$	$10^{-4}\sigma$	$10^{-3}\sigma$	$10^{-2}\sigma$	$10^{-1}\sigma$	σ	10σ	$10^{2}\sigma$	$10^{3}\sigma$	$10^{4}\sigma$	$10^{5}\sigma$	$10^{6}\sigma$	
Unormalized	$k=1$	54.78	49.08	49.08	48.08	48.08	48.10	48.10	48.10	48.13	48.13	48.13	48.13	48.09	48.10	55.38
	$k=4$	59.05	54.69	54.69	54.69	54.69	54.62	54.60	54.61	54.15	54.15	54.15	54.18	54.22	54.15	59.80
	$k=32$	54.66	51.37	51.37	51.37	51.37	51.52	51.52	51.50	51.51	51.51	51.78	51.78	51.78	51.75	55.31
Normalized	$k=1$	55.10	49.23	49.23	49.05	49.11	49.11	49.12	49.12	49.12	49.12	49.12	49.11	49.11	49.11	55.95
	$k=4$	59.2	54.89	54.89	54.89	54.60	54.62	53.95	53.95	53.95	53.95	53.95	53.96	53.96	53.96	59.98
	$k=32$	54.90	50.46	50.46	50.45	50.10	50.10	50.11	50.10	50.10	50.12	50.12	50.10	50.10	50.10	55.70
Random w	$k=1$	59.78	56.71	56.71	56.71	56.77	56.71	56.71	56.71	56.74	56.66	56.66	56.66	56.66	56.68	60.10
	$k=4$	61.25	56.80	56.80	56.80	56.80	56.80	56.75	56.76	56.75	56.70	56.70	56.70	56.70	56.72	61.35
	$k=32$	59.95	56.74	56.74	56.74	56.74	56.74	56.74	56.74	56.76	56.68	56.68	56.68	56.65	56.65	61.16
Multi-lap		61.50	57.00	56.95	56.93	56.93	56.93	56.90	56.96	56.91	56.91	56.94	56.94	56.95	56.97	62.70

Table 6: Performances on UCF for different elementary laplacians (normalized, unormalized and random walk) and their marginal and total combinations using MLGCN (note that our expansion is not used). In this table, "binary" means that $\mathbf{A}^k$ is used to build the elementary laplacian while "binary $\times$ gaussian" means that “$\mathbf{A}^k \times$ gaussian similarity’ is used instead; for each graph $\mathcal{G}$, the scale σ of the gaussian similarity is taken as the average distance between node features in $\mathcal{G}$.