Higher order Dictionary Learning for Compressed Sensing based Dynamic MRI reconstruction

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Abstract

Compressed sensing (CS) is one of the predominant tools used presently to explore the possibilities in accelerating cardiac and body Magnetic Resonance (MR) imaging for achieving shorter scans and accommodating a wider patient group. CS accomplishes this by manoeuvring the scanning hardware to make much fewer measurements and imposing sparsity of the MR image in a known basis on the reconstruction process. Prior works have adopted fixed transforms as well as dictionaries learned on image patches to incorporate the sparsifying basis. Despite the obvious merits of 1-D dictionary learning methods, they suffer from high computational and memory complexity when extended to higher patch sizes and are thus restricted to capturing only local features. Thus, there arises the need for an efficient framework to extend the advantages of dictionary learning to higher dimensional applications like dynamic MRI (dMRI) where there exists a strong correlation across successive frames. This work employs a tensor decomposition based dictionary learning approach to effectively extend CS to dMRI and exploit the temporal gradient (TG) sparsity, thereby retaining maximum spectral and temporal resolution at higher under-sampling factors. The proposed technique is experimentally validated to achieve significant improvement in reconstruction quality over the current state-of-art in dynamic MRI under distinct sampling trajectories and noisy conditions. Further, it facilitates faster reconstructions of the dMRI volume than existing methods, rendering it an ideal choice in critical scenarios which demand a swift diagnosis.

1 Introduction

Magnetic Resonance Imaging (MRI) is an important diagnostic modality in medical imaging with unparalleled ability to provide soft tissue contrast using non-invasive acquisition techniques. A major challenge in MRI is the long acquisition time for which the patient is required to stay motionless. Parallel imaging techniques like Generalized Auto-calibrating

Partially Parallel Acquisitions (GRAPPA) [5] and SENSitivity Encoding (SENSE) [8] have been incorporated into modern MRI systems to considerably reduce imaging time by reducing the number of phase encoding steps during acquisition. However, these techniques can attain a maximum of twofold acceleration due to imperfect aliasing corrections. An alternate technique to accelerate MRI is to under-sample the k-space and then use the theory of Compressed Sensing(CS) [3] to recover the original image. MR data is traditionally acquired either slice wise (2D imaging ie 2D k-space data acquisition) or volume wise (3D imaging ie 3D k-space acquisition). However, reconstruction of the under-sampled k-space data is mostly carried out slice wise in the literature and is referred to as Static MRI reconstruction. For successful recovery, CS theory requires incoherent sampling and the prior knowledge of the basis in which the MR image is sparse. While random under-sampling of the k-space [7] using suitable trajectories meets the first requirement, the latter consists of finding suitable sparsifying bases for MR images. Dictionaries learned on vectorized patches of the image have been found to outperform fixed transforms in capturing essential features. Some works like [9] and [14] learn dictionaries on 8×8 patches of each under-sampled slice to reconstruct the MR image. A joint label consistent embedding and dictionary learning approach is used in classification in [15]. One can also see other dictionary learning approaches like analysis mechanism based structured dictionary learning and discriminative Fischer embedding dictionary learning algorithms being used in works [16] and [15] respectively. As these methods are restricted to learn dictionaries on small patches, they forego prior knowledge of the anatomy of the specific image sequence. In order to leverage this information, deep learning approaches like Deep De-Aliasing Generative Adversarial Networks (DAGAN) [13] attempt to improve recovery by learning from large training data sets unlike conventional dictionary learning based schemes. To imitate the learning capabilities of deep learning based methods at lower computational costs, there have been concerted efforts [12] to learn higher order dictionaries by employing tensor decomposition methods.

In contrast to the static MR regime discussed above, dynamic MRI focuses on a particular region of the body and perform the reconstruction over time. Dynamic MRI concentrates on studying the changing behavior in human body like blood flow, heartbeat, and changes in concentration like BOLD (blood oxygen level-dependent) signals. Let x_t represent a MRI frame at t^{th} instant. Let each image pixel be of size $\sqrt{P} \times \sqrt{P}$ and T be the total number of frames collected. Let S_t be the k-space data for the t^{th} frame. The problem associated with dynamic MRI is to recover all x_t's $(t = 1 \ldots T)$ from the collected k-space data S_t's. The MR imaging equation for each frame is as follows:

$$S_t = F_u x_t \tag{1}$$

where F_u is the under-sampled forward Fourier operator.
In dynamic MRI, each of the frames is spatially correlated. Thus, these frames are sparse in transform domains like wavelets and also have small total variations. Dynamic MRI sequence is temporally correlated; this is because temporal changes are slow and one frame looks similar to the previous one. Thus, in dynamic MRI, the inter-frame temporal correlation should also be exploited along with intra-frame spatial correlations in order to achieve the best possible reconstruction.

The sparsity in both temporal and spatial domains are exploited in various dynamic MRI reconstruction methods. The spatio-temporal resolution was improved by introducing various model based image reconstruction schemes. The sparsity of data in *x-f* space is exploited to recover the dynamic images from under-sampled measurements.However the performance of this method is very disappointing due to patient motion [4]. Some works use the compact representation of data in the Karhunen Louve transform (KLT) domain by posing the problem as a spectrally regularized matrix recovery problem [6]. The fixed based transforms are now replaced by adaptive transforms obtained from dictionary learning (DL) for the two-dimensional structural MRI reconstruction. Sparsity is then enforced on the temporal gradient (TG) of the reconstruction [1]. An extension of conventional total variation known as the dynamic total variation (dTV) is used for exploiting the temporal and spatial sparsity. This method uses an accelerated re-weighted least squares algorithm to solve the reconstruction problem [11].

In this paper, an improved 2D separable dictionary learning algorithm is proposed to address memory and computation costs for dynamic MR image reconstruction by exploiting the Kronecker structure of the 3D under-sampled k-space data. Dictionary learning is unraveled by the CANDECOMP/PARAFAC (CP) tensor decomposition which is a generalization of singular value decomposition (SVD) from 2D matrices to tensors. The temporal gradient(TG) sparsity of the dynamic MRI data is also added as a constraint in the dictionary learning problem. This formulation achieves significant gain in memory and computation for learning the higher order dictionary. The algorithm is found to have superior reconstruction quality in noiseless and noisy k-space acquisition scenarios for random, cartesian and radial sampling trajectories. The paper is organized mainly into three sections. Section 2 describes the preliminaries and notations used in this paper, Section 3 explains the proposed method and Section 4 highlights the experimental results.

2 Preliminary

A tensor may be represented as a multidimensional array. Just as a vector in an n-dimensional space is represented by a one-dimensional array of length n with respect to a given basis, any tensor with respect to a basis is represented by a multidimensional array. A first-order tensor is a vector, a second-order tensor is a matrix, and tensors of order three or higher are called higher-order tensors. A matrix or vector is represented by bold font. Tensors are denoted by bold face Euler script letters, $\mathcal{X} \in \mathbb{R}^{I\times J\times K}$. $\circ$, $\odot$ and $\otimes$ represent outer product, Khatri-Rao product and Kronecker product respectively.

3 Proposed Method

Generally a dynamic MRI sequence is acquired as a three dimensional volume $\mathcal{G}$ of size $I = I_1 \times I_2 \times I_3$, with the third dimension indicating time. In dynamic MRI, each of the frames is spatially correlated like static MRI. Thus, these frames are sparse in transform domains like wavelets and also have small total variations. Another property dynamic MRI is that the MRI sequence is temporally correlated, this is because temporal changes are slow and one frame looks similar to the previous one. Thus in dynamic MRI inter-frame temporal correlation should be exploited along with intra-frame spatial correlations in order to achieve the best possible reconstruction.

The proposed method accepts the 3D under-sampled MRI data $\bar{\mathcal{G}}$ of size $I = I_1 \times I_2 \times I_3$ as input. The primary step involves computing the zero filled inverse FFT of each frame to obtain the tensor of zero-filled images $\mathcal{G}$ with under-sampling artifacts. The dictionaries $\hat{D}_1$ and $\hat{D}_2$ are initialized to the product of a random sensing matrix and the wavelet basis. In the sparse coding stage, N-BOMP computes the sparse representation from the zero-filled tensor $\mathcal{G}$ and dictionaries $\hat{D}_1$ and $\hat{D}_2$ and outputs an approximate tensor $\mathcal{B}$. The dictionary learning stage is accomplished by employing CP decomposition on the residual error ζ obtained after processing the approximated tensor $\mathcal{B}$. CP decomposition is done using the adapted Alternating Least Squares(ALS) algorithm [10]. The dictionaries $\hat{D}_1$ and $\hat{D}_2$ are then updated along with the simultaneous updation of Temporal gradient(TG) and the cycle of sparse coding and dictionary update repeats. The ratio of γ_{k+1}/γ_k or maximum number of iterations n is used to determine termination of the algorithm.The pseudo code of the proposed algorithm is given in Algorithm 1.

Algorithm 1 Proposed Dictionary Learning Algorithm

Input: $\bar{\mathcal{G}}$
Output: $\hat{\mathcal{P}}$

Compute $\mathcal{G} = IFFT(\bar{\mathcal{G}})$ along mode-1 and mode-2
Initialize $\hat{D}_1 = \Phi_1 W_1$,$\hat{D}_2 = \Phi_2 W_2$ and $k = 0$
Solve $\mathcal{B}$ using N-BOMP
for j=1 *to n* **do**
    Compute $\zeta^{j,j}$
    Do CP decomposition using ALS algorithm on $\zeta^{j,j}$ using Equation 12
    Update $\hat{D}_1$,$\hat{D}_2$ and $\hat{\mathcal{P}}$ using Equation 9
end for
$\gamma_{k+1} = \min_{\hat{\mathcal{P}},\hat{D}_1,\hat{D}_2} ||\mathcal{G} - \hat{\mathcal{P}} \otimes_1 \hat{D}_1 \otimes_2 \hat{D}_2|| + TG\{\mathcal{G}\}$
$k = k+1$
while $(\gamma_{k+1}/\gamma_k > 1)$ **do**

3.1 Problem Formulation

Given a 3-way dynamic MRI tensor $\mathcal{G} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, the linear structure of the unfolding matrix of this tensor can be exploited by using Tucker decomposition. It is a form of higher order PCA. It decomposes a tensor into a *core tensor* multiplied by a matrix along each mode. The Tucker decomposition is as follows:

$$\mathcal{G} = \mathcal{Y} \times_1 D_1 \times_2 D_2 \times_3 D_3 \tag{2}$$

where $\mathcal{Y} \in \mathbb{R}^{M_1 \times M_2 \times M_3}$ is the core tensor and $D_n \in \mathbb{R}^{I_n \times M_n}$ is the factor matrices or dictionaries. Tucker model can be represented in the form of Kronecker product as:

$$g = (D_3 \otimes D_2 \otimes D_1)y \tag{3}$$

Here g and y are the vectorized versions of 3-way MRI tensor $\mathcal{G}$ and *core tensor* $\mathcal{Y}$ respectively. Factor matrices also known as dictionaries are defined by $D_n = \Phi_n W_n$ where $\Phi_n \in \mathbb{R}^{I_n \times M_n}$ and W is some basis (it can be wavelet,contourlet and so on). Let $H_n \in \mathbb{R}^{I_n \times S_n}$ be the sub-matrices obtained by restricting the mode-n dictionaries to the columns indicated by support indices $\mathcal{I}_n$ $(H_n = D_n(:,\mathcal{I}_n))$, then this signal can be approximated in vector form as:

$$\hat{g} = (H_3 \otimes H_2 \otimes H_1)b \tag{4}$$

where $b \in \mathbb{R}^K$ $(K = \prod_{n=1}^{3} S_n)$ is the vectorised form of N-way block array consisting of only non-zero entries.From this, the general minimization problem is:

$$b = arg \min_r ||(H_3 \otimes H_2 \otimes H_1)r - g||_2^2 \tag{5}$$

where $g \in \mathbb{R}^{I_1 I_2 I_3}$ is the vectorised version of $\mathcal{G}$.The solution to this problem is given by $b = [H^T H]^{-1} Hg$ which means that $[H^T H]b = Hg$. The N-BOMP is shown in [2] to be less complex having $\mathbb{O}(S)$ complexity against $\mathbb{O}(S^N)$ in case of classical OMP and K-OMP, where S implies the block sparsity of the N-dimensional tensor.

The pseudo code of the N-BOMP algorithm is given in Algorithm 2.

Algorithm 2 N-BOMP Algorithm Adaptation

Require: mode-3 dictionaries $\{D_1, D_2, D_3\}$ with $D_n \in \mathbb{R}^{I_n \times M_n}$,signal $\mathcal{G} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, maximum number of non-zero entries K_{max}, tolerance ε

Ensure: Sparse representation $\mathcal{G} \approx \mathcal{Y} \times_1 D_1 \times_2 D_2 \times_3 D_3$

$\mathcal{I}_n = [\emptyset]$,$\mathcal{Q} = \mathcal{G}$,$\mathcal{Y} = 0$,$k = 1$

$N \leftarrow length(B)$

while $|\mathcal{I}_1||\mathcal{I}_2||\mathcal{I}_3| \leq K_{max}$ *and* $||\mathcal{Q}||_F \geq \varepsilon$ **do**

  $[i_1^k i_2^k i_3^k] = \arg\max_{[i_1 i_2 i_3]} |\mathcal{Q} \times_1 D_1^T(:, i_1) \times_2 D_2^T(:, i_2) \times_3 D_3^T(:, i_3)|$

  $b = \arg\min_r ||(H_3 \otimes H_2 \otimes H_1)r - g||_2^2$

  $\mathcal{Q} = \mathcal{G} - \mathcal{B} \times_1 H_1 \times_2 H_2 \times_3 H_3$

  $k = k + 1$

  return $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\}$, $\mathcal{B}$

The output of N-BOMP is an approximated tensor $\mathcal{B}$. This tensor is then subjected to CP decomposition after computing the residual error ζ. Approximated tensor B_i is decomposed into $\hat{D}_1 \hat{P}_i \hat{D}_2^T$ where $\hat{P}_i$ is sparse, which deals with spatial sparsity.
Temporal gradient sparsity (TG) is exploited as an extra constraint in sparse reconstructions under the assumption that medical images are piecewise smooth.

$$TG\{\mathcal{G}\} = \sqrt{\bigtriangledown_{I_1}\{\mathcal{G}\}^2 + \bigtriangledown_{I_2}\{\mathcal{G}\}^2 + \bigtriangledown_{I_3}\{\mathcal{G}\}^2} \tag{6}$$

where $\bigtriangledown_{I_1}$ is the gradient along I_1^{th} mode. Hence the objective function can be formulated as:

$$\gamma = \min_{\hat{\mathcal{P}}, \hat{D}_1, \hat{D}_2} ||\mathcal{G} - \hat{\mathcal{P}} \otimes_1 \hat{D}_1 \otimes_2 \hat{D}_2|| + TG\{\mathcal{G}\} \tag{7}$$

where $\hat{\mathcal{P}} \in \mathbb{R}^{I_n \times M_n \times l}$ and $\hat{D}_n \in \mathbb{R}^{I_n \times M_n}$ is the approximated dictionary after N-BOMP, then $\mathcal{G}$ can be formulated as:

$$\mathcal{G} = (d_j^1 \circ d_k^2 \circ p_{j,k}) + \sum_{a \neq j}^{N} (d_j^1 \circ d_a^2 \circ p_{j,a}) + \sum_{a \neq j}^{N} \sum_{b \neq k}^{N} (d_a^1 \circ d_b^2 \circ p_{a,b}) + TG\{\mathcal{G}\} \tag{8}$$

where d_j^1 and d_k^2 are the jth and kth atoms of D_1 and D_2 respectively and $p_{j,k}$ is the corresponding sparse representation vector . Then the corresponding residual

$$\begin{aligned}\zeta^{j,k} &= \mathcal{G} - \sum_{a\neq j}^{N}\sum_{b\neq k}^{N}(d_a^1 \circ d_b^2 \circ p_{a,b}) + TG\{\mathcal{G}\} \\ &= (d_j^1 \circ d_k^2 \circ p_{j,k}) + \sum_{a\neq j}^{N}(d_j^1 \circ d_a^2 \circ p_{j,a}) + TG\{\mathcal{G}\}\end{aligned} \tag{9}$$

d_j^1, d_k^2 and $p_{j,k}$ can be solved by performing CP decomposition using ALS algorithm on $\zeta^{j,k}$.

The CP decomposition factorizes the tensor ζ into a sum of component rank-one tensors.The third order tensor $\zeta \in \mathbb{R}^{I_1\times I_2\times I_3}$ can be decomposed as:

$$\zeta \approx \sum_{r=1}^{R} e_r \circ f_r \circ g_r \tag{10}$$

where R is a positive integer. The columns I_1,I_2 and I_3 are normalized to unit length with weights absorbed to vector $\eta \in \mathbb{R}^R$ so that:

$$\zeta \approx [\![\eta; E, F, G]\!] \equiv \sum_{r=1}^{R} \eta_r e_r \circ f_r \circ g_r \tag{11}$$

CP decomposition of ζ is achieved by employing the ALS method. Computing a CP decomposition that best approximates ζ results in the following minimization problem:

$$\begin{aligned}\min_{\hat{\zeta}} ||\zeta - \hat{\zeta}|| \quad with \quad \hat{\zeta} &= [\![\eta; E, F, G]\!] \\ &\equiv \sum_{r=1}^{R} \eta_r e_r \circ f_r \circ g_r\end{aligned} \tag{12}$$

The ALS approach fixes F and G to solve for E so the problem reduces to a linear least-squares problem.At each inner iteration, the pseudo inverse for a matrix Z must be calculated as given in Algorithm 3.

Algorithm 3 CP-ALS Algorithm Adaptation

Initialize $E^{(n)} \in \mathbb{R}^{I_n\times R}$ for $n = 1, 2, .., N$
1: **repeat**
2: **for** $n = 1$ to N **do**
3: $Z \leftarrow E^{(1)T}E^{(1)} * ... * E^{(n-1)T}E^{(n-1)} * E^{(n+1)T}E^{(n+1)} * ... * E^{(N)T}E^{(N)}$
4: $E^{(n)} \leftarrow \zeta^{(n)}(E^{(N)} \odot ... \odot E^{(n-1)} \odot E^{(n+1)} \odot ... E^{(1)})Z^{\dagger}$
5: Normalize columns of $E^{(n)}$ (Storing norms as η)
6: **until** fit seems to improve or iterations exhausted
7: **return** η ,$E^{(1)}, E^{(2)}, ..., E^{(N)}$

The learned dictionaries can be used to recover new zero-filled image tensors of the scan type by simply employing the N-BOMP algorithm. For other scan types, the proposed dictionary learning algorithm can be adopted to learn efficient dictionaries corresponding to the specific scan type. The proposed method yields superior reconstruction over the state-of-art due to learning dictionaries of size relative to image size and not restricting itself to patches and by also exploiting the temporal gradient(TG) sparsity.

4 Experimental Results

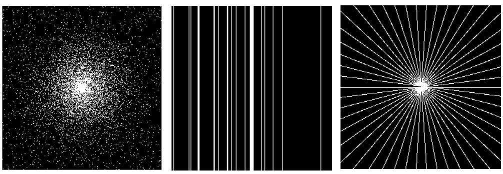
In-vivo breath-hold cardiac perfusion dataset is used for this experiment. It is of size $256 \times 256 \times 70$. Cartesian, 2D-random and radial under-sampling schemes are investigated to thoroughly examine the performance of the algorithm. The performance is compared with the state-of-art dynamic MRI reconstruction algorithms K-t SLR [6] and dTV [4], and also with the superior DLTG algorithm [1], each set to their respective optimum parameters. The performance criteria used here are PSNR (Peak Signal-to-Noise ratio) and SSIM(Structural Similarity Index). PSNR is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. PSNR in dB is expressed as:

$$PSNR = 20log_{10}\frac{MAX_i}{\sqrt{MSE}} \tag{13}$$

where MAX_i is the maximum possible pixel value of the image and MSE is the Mean Square Error. SSIM is designed to improve on traditional methods such as peak signal-to-noise ratio (PSNR) and mean squared error (MSE).

4.1 Noiseless Case

To evaluate the performance of the proposed method under assumptions of noiseless acquisition, the PSNR and SSIM parameters are compared with dynamic reconstruction algorithms like K-t SLR [6], dTV [4] and DLTG [1]. This noiseless acquisition is done for Gaussian mask, Cartesian mask and Radial mask respectively since dynamic MRI frames are reconstructed by adopting retrospective sampling in the k-space of each frame. The Gaussian, Cartesian and Radial masks corresponding to 10% under-sampling is given in Figure 1.



(a) Gaussian Mask (b) Cartesian Mask (c) Radial Mask

Figure 1: Gaussian Mask, Cartesian Mask and Radial Mask corresponding to 10% under-sampling

4.1.1 Gaussian Mask

The k-space of each frame is multiplied by Gaussian mask and then the proposed algorithm is carried out. The PSNR and SSIM comparison of various methods when the MRI frame is multiplied with a Gaussian mask corresponding to 1%, 5%, 10%, 20%, 30%, 40% and 50% under-sampling is given in Figure 3. It is observed from the figure 3 that the proposed method has better PSNR and SSIM numbers than the state-of-the-art.

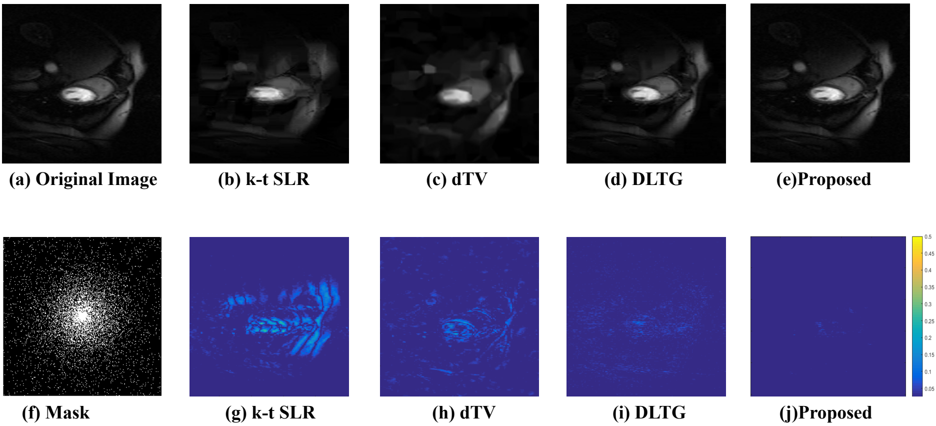


Figure 2: Figure (a)-(e) shows Reconstructed frames, Figure (f) shows the Gaussian Mask and Figure (g)-(h) shows Reconstruction errors for Zero-filled Image, k-t SLR[6], dTV[4], DLTG[1] and Proposed Method

4.1.2 Cartesian Mask

The k-space of each frame is multiplied with a Cartesian mask and proposed algorithm is carried out. The dynamic MRI frame is multiplied with Cartesian mask corresponding to 10%, 20%, 30%, 40% and 50% under-sampling and the PSNR and SSIM comparisons are shown in Figure 4. It is evident from the graphs that the proposed methods show better results.

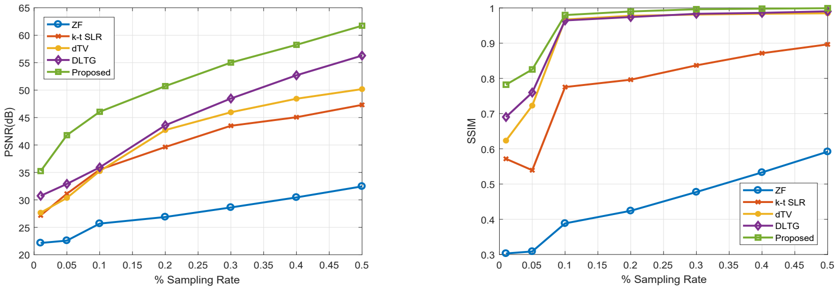


Figure 3: Comparison of PSNR and SSIM using Gaussian Mask

4.1.3 Radial Mask

Radial mask is used in this case and proposed algorithm is implemented on the dynamic MRI frame afterwards. For instance, when frame 24 is considered it has 9 dB, 7 dB and 5 dB improvement in PSNR when compared to k-t SLR [6], dTV [4] and DLTG [1] respectively. Similar improvement is also seen in SSIM numbers from the simulation results.

It is evident from the simulation results that the structural information in the reconstructed results is far better while using tensor based dictionary as compared to vector based

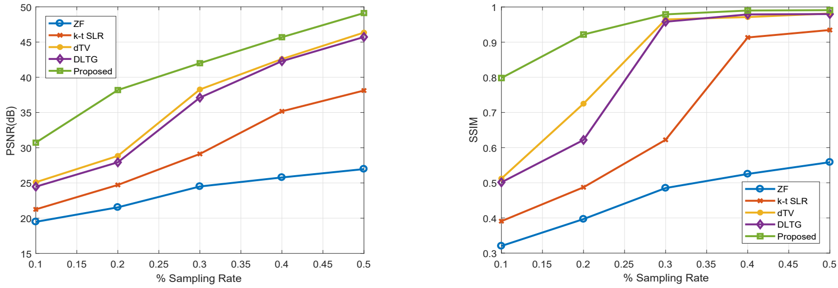


Figure 4: Comparison of PSNR and SSIM using Cartesian Mask

dictionary. Both the temporal correlation and spatial structures are exploited by making use of sparsity constraints imposed on the tensor dictionary.

4.2 Noisy Case

To evaluate performance under noisy acquisition, an additive Gaussian noise with zero mean and variance σ^2=18.8 is added to the k-space of each MRI frame (In this case Gaussian mask is used). The corresponding zero-filled images are generated and passed to the algorithms. This experimental setup would help perceive the robustness of the proposed algorithm. The noise robustness of the proposed method is given in Figure 5, which clearly depicts the superior performance of the proposed method over the state-of-art. The visual comparison of proposed method over state-of-the- art methods in noisy reconstruction scenario is shown in Figure 6. It is because in tensor dictionary each dictionary atom preserves the structural signature in the image and hence the noise in the image is suppressed. The performance under noisy scenario is also evaluated using Cartesian and Radial masks. The experimental results show that the proposed method has 9dB, 8dB and 7dB increase in PSNR over k-t SLR, dTV and DLTG respectively for Cartesian mask. For Radial mask, the proposed method shows 9dB, 6dB and 4dB improvement in PSNR over k-t SLR, dTV and DLTG respectively. Similar improvement is also seen in SSIM numbers for both the masks.

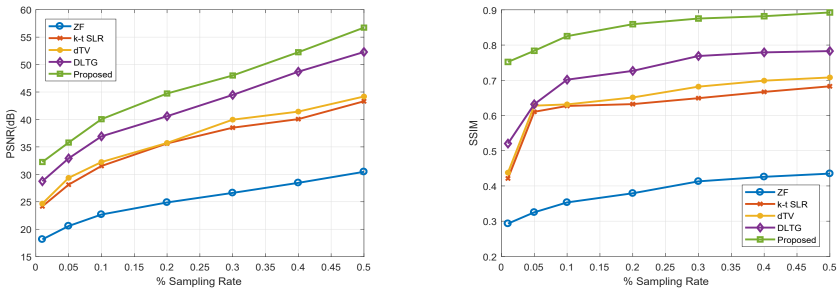


Figure 5: Noise Robustness

Another advantage of the proposed method is the time taken for reconstruction. The overall dictionary learning step and joint reconstruction for 70 frames take approximately 10 minutes. It is observed that this same dictionary can be used to jointly reconstruct 30 distinct

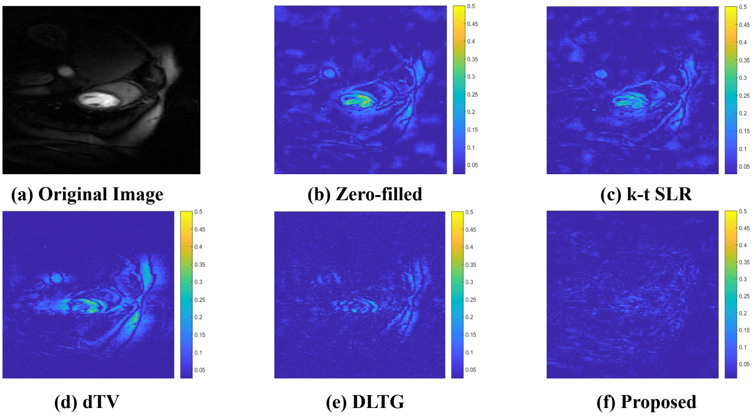


Figure 6: Figure (a) shows Original Image and Figures (b)-(f) shows Reconstruction errors of Zero-filled, k-t SLR [6], dTV [4], DLTG [1] and Proposed Method respectively

images of similar scan type using N-BOMP in 30 seconds, that is one frame is reconstructed in one second. Whereas, the corresponding reconstructions of each slice by the state-of-art algorithms k-t SLR, dTV and DLTG take roughly 100, 1.42 and 10 seconds respectively. Thus, the proposed method is found to achieve faster and superior reconstructions over its counterparts.

5 Conclusion

This work proposes a tensor based dictionary learning algorithm which exploits the Kronecker structure dynamic MRI data to efficiently learn separable dictionaries. The learned dictionaries are compact and the proposed method is a novel reconstruction strategy that exploits the temporal and spatial redundancies of the dynamic MRI data and accelerates the reconstruction time. The proposed method outperforms the current state-of-art in Compressed Sensing MRI in both the noiseless and noisy k-space acquisition scenarios for different under-sampling factors. This method also exploits the temporal gradient(TG) sparsity in order to achieve the best possible reconstruction.

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